LESSON PLANS

DATE:

SUBJECT: PreCalc

TOPIC: Sec 10-2 matrices

OBJECTIVES:

* Students are able to write the augmented matrix of a system or a system from an augmented matrix
* Students are able to perform row operations and solve systems using matrices

PROCEDURE:

Warm-up → SOLVE the system ALGEBRAICALLY

$$\begin{cases} 2x-y+z=0 \\ x+z=1 \\ x+2y=8 \end{cases}$$ > eliminate z

$$\begin{array}{r} 2x-y+z=0 \\ -x\quad\quad -z=-1 \\ \hline x-y=-1 \\ -x-2y=-8 \\ \hline -3y=-9 \end{array}$$ $y=3$

$x+6=8$ $x=2$

$2+z=1$ $z=-1$

* easier solved using matrices

Matrix → a rectangular array of numbers

$$\begin{array}{c|cccc} & \text{COLUMN 1} & \text{COLUMN 2} & \cdots & \text{COLUMN } n \\ \text{ROW 1} & a_{11} & a_{12} & \cdots & a_{1n} \\ \text{ROW 2} & a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & & \\ \text{ROW } m & a_{m1} & a_{m2} & \cdots & a_{mn} \end{array}$$

called entries

a_{mn}: row × column

DIMENSION: m rows by n columns ($m \times n$)

augmented matrix → used to represent a system using only coefficients

ASSIGNMENT:

ex) WRITE the augmented matrix

a) $3x-4y=-6$
$2x-3y=-5$

★ variable terms on left
★ constants on right

$$\left[\begin{array}{cc|c} 3 & -4 & -6 \\ 2 & -3 & -5 \end{array}\right]$$ (dimensions are 2×3)

b) $4x-3y+z=0$
$x+z-5=0$
$x+2y-2z=1$

$$\left[\begin{array}{ccc|c} 4 & -3 & 1 & 0 \\ 1 & 0 & 1 & 5 \\ 1 & 2 & -2 & 1 \end{array}\right]$$

c) Given the augmented matrix, write the system.

$$\left[\begin{array}{ccc|c} 3 & -1 & -1 & 7 \\ 2 & 0 & 2 & 8 \\ 0 & 1 & 1 & 0 \end{array}\right] \rightarrow \begin{cases} 3x-y-z=7 \\ 2x+2z=8 \\ y+z=0 \end{cases}$$

coefficient matrix

USE ROW OPERATIONS TO SOLVE SYSTEMS USING MATRICES WITHOUT CALC.

ROW OPERATIONS:

1) you may interchange any two rows
2) you may replace a row by a nonzero multiple of that row
3) you may replace a row by the sum of that row and a constant nonzero multiple

ex) $\left[\begin{array}{cc|c} 1 & -2 & 2 \\ 3 & -5 & 9 \end{array}\right]$ apply $R_2=-3r_1+r_2$ ★ r_1 remains unchanged

$$\left[\begin{array}{cc|c} 1 & -2 & 2 \\ 0 & 1 & 3 \end{array}\right]$$

$-3\times1+3$ $\quad$ $-3\times-2+5$ $\quad$ $2\times-3+9$

Row Echelon Form

$$\begin{bmatrix} 1 & a & b & | & d \\ 0 & 1 & c & | & e \\ 0 & 0 & 1 & | & f \end{bmatrix}$$

must be 1

entries under main diagonal = 0

Main diagonal = 1

★ any rows that contain all zeros to the left of the vertical bar appear at the bottom

1x) SOLVE USING MATRICES WITHOUT CALC

$$\begin{cases} 2x + 2y = 6 \\ x + y + z = 1 \\ 3x + 4y - z = 13 \end{cases}$$

must be 1

$$\begin{bmatrix} 2 & 2 & 0 & | & 6 \\ 1 & 1 & 1 & | & 1 \\ 3 & 4 & -1 & | & 13 \end{bmatrix}$$ ← switch rows

$$\begin{bmatrix} 1 & 1 & 1 & | & 1 \\ 2 & 2 & 0 & | & 6 \\ 3 & 4 & -1 & | & 13 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & | & 1 \\ 0 & 0 & -2 & | & 4 \\ 0 & 1 & -4 & | & 10 \end{bmatrix}$$ ← switch

make 0

$-2r_1 + r_2$

$-3r_1 + r_3$

$$\begin{bmatrix} 1 & 1 & 1 & | & 1 \\ 0 & 1 & -4 & | & 10 \\ 0 & 0 & -2 & | & 4 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & | & 1 \\ 0 & 1 & -4 & | & 10 \\ 0 & 0 & +1 & | & -2 \end{bmatrix}$$ ← Mult by $-\frac{1}{2}$

Now create system

$z = -2$

$y + -4z = 10$

so $y + 8 = 10$

$y = 2$

$x + y + z = 1$

$x + 2 + -2 = 1$

$x = 1$

★ WITH CALC

use rref (reduced row echelon form)

(B)

$$\begin{bmatrix} 1 & 0 & 0 & | & a \\ 0 & 1 & 0 & | & b \\ 0 & 0 & 1 & | & c \end{bmatrix} \begin{matrix} \rightarrow x = a \\ \rightarrow y = b \\ \rightarrow z = c \end{matrix}$$

Check above problem →

$$\begin{bmatrix} 1 & 0 & 0 & | & 1 \\ 0 & 1 & 0 & | & 2 \\ 0 & 0 & 1 & | & -2 \end{bmatrix}$$

$x = 1$

$y = 2$

$z = -2$

1) Store in a 3x4 Matrix [A]

2) Matrix → Math

rref [A]

ex) $\begin{cases} x+y+z=6 \\ 2x-y-z=3 \\ x+2y+2z=0 \end{cases}$ $\left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 2 & -1 & -1 & 3 \\ 1 & 2 & 2 & 0 \end{array}\right]$

$\left[\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]$ → $x=0$, $y+z=0$, $0z=1$ → NOT POSSIBLE → INCONSISTENT

ex) $\begin{cases} 6x-y-z=4 \\ -12x+2y+2z=-8 \\ 5x+y-z=3 \end{cases}$ $\left[\begin{array}{ccc|c} 6 & -1 & -1 & 4 \\ -12 & 2 & 2 & -8 \\ 5 & 1 & -1 & 3 \end{array}\right]$ →

change dec to fraction

$\left[\begin{array}{cccc} 1 & 0 & -2/11 & 7/11 \\ 0 & 1 & -1/11 & -2/11 \\ 0 & 0 & 0 & 0 \end{array}\right]$ → $0=0$ ALWAYS true dependent

$x - \frac{2}{11}z = \frac{7}{11}$ → $x = \frac{2}{11}z + \frac{7}{11}$

$y - \frac{1}{11}z = -\frac{2}{11}$ → $y = \frac{1}{11}z - \frac{2}{11}$

z = any real #

$\begin{cases} x+y+z+w=4 \\ 2x-y+z=0 \\ 3x+2y+z-w=6 \\ x-2y-2z+2w=-1 \end{cases}$ $\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 4 \\ 2 & -1 & 1 & 0 & 0 \\ 3 & 2 & 1 & -1 & 6 \\ 1 & -2 & -2 & 2 & -1 \end{array}\right]$

4x5

$\left[\begin{array}{ccccc} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{array}\right]$

$x=1$, $y=2$, $z=0$, $w=1$